Basic Algebraic Formulas

- $(a+b)^2 = a^2+b^2+2ab$
- $(a-b)^2 = a^2+b^2-2ab$
- $a^2-b^2 = (a+b)(a-b)$

- $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$
- $(a+b)^2-(a-b)^2 = 4ab$

- $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

  oR $= a^2+b^2+c^2+2(ab+bc+ca)$
- $a^2+b^2 = (a+b)^2-2ab$

  oR

  $a^2+b^2 = (a-b)^2+2ab$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$(a+b)^3 = a^3+3ab(a+b)+b^3$

$(a-b)^3 = a^3-3ab(a-b)-b^3$

$\left[x+\frac{1}{x}\right]^3 = x^3+3\left[x+\frac{1}{x}\right]+\frac{1}{x^3}$

$\left[x-\frac{1}{x}\right]^3 = x^3-3\left[x-\frac{1}{x}\right]-\frac{1}{x^3}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

There are two roots of Quadratic formula

- $$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
- $$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$
- $$\alpha + \beta = \frac{-b}{a}$$
- $$\alpha - \beta = \frac{c}{a}$$

Note:

Main part of quadratic formula, $b^2 - 4ac$ is called **Discriminant**. The nature of these roots depends on the value of $b^2 - 4ac$

Properties of Real Number

- Closure propert w.r.t addition.

  $\forall a, b \in R \Rightarrow a+b \in R$

Example:

-5 and $11 \in R$

$-5+11 = 6 \in R$

- Closure property w.r.t Multiplication

  $\forall a, b \in R \Rightarrow a.b \in R$

Example:

-5 and $11 \in R$

$(-5)\times(11) = -55 \in R$

- Commutative property w.r.t addition

  a and $b \in R$

then $a+b = b+a$

Example

13 and $7 \in R$

$13+7 = 7+13$

- Commutative property w.r.t Multiplication

  a and $b \in R$

  $a \times b = b \times a$

Example

13 and $7 \in R$

$13 \times 7 = 7 \times 13$

- Associative property w.r.t addition

  a, b and $c \in R$

  $a+(b+c) = (a+b)+c$

Example:

$5, 4$ and $7 \in R$

$5+(4+7) = (5+4)+7$

- Associative property w.r.t multiplication

  a, b and $c \in R$

  $a \times (b \times c) = (a \times b) \times c$

Example:

$5, 4$ and $7 \in R$

$5 \times (4 \times 7) = (5 \times 4) \times 7$

- Additive Identity property

  if $a \in R$, there is a unique number zero (0) called additive Identity

  $a+0 = 0+a = a$

Example:

$2+0 = 0+2 = 2$

- Multiplicative Identity property

  if $a \in R$, there is a unique number one (1) called multiplicative Identity.

  $a \times 1 = 1 \times a = a$

Example:

$$3 \times 1 = 1 \times 3 = 3$$

- Additive Inverse property

if $a \in R$, there exists a unique real number $-a$ called additive Inverse.

$$a + (-a) = 0 = -a + a$$

Example:

Additive Inverse of 8 is -8

$$8 + (-8) = 0 = -8 + 8$$

- Multiplicative Inverse property

if $a \neq 0 \in R$, there exists a unique real number $\frac{1}{a}$ called multiplicative Inverse

$$a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$$

Example:

7 and $\frac{1}{7}$ are multiplicative inverse of each other.

$$7 \times \frac{1}{7} = 1 = \frac{1}{7} \times 7$$

- Distributive property of Multiplication over addition.

a, b and $c \in R$

$a(b+c) = ab + ac$

$(a+b)c = ac + bc$

Example:

$5, 6$ and $7 \in R$

$5 \times (6+7) = 5 \times 6 + 5 \times 7$

$(5+6) \times 7 = 5 \times 7 + 6 \times 7$

- Distributiv property of Multiplicative over Subtraction:

a, b and $c \in R$

$a(b-c) = ab - ac$

$(a-b)c = ac - bc$

Example:

$5, 6$ and $7 \in R$

$5 \times (6-7) = 5 \times 6 - 5 \times 7$

$(5-6) \times 7 = 5 \times 7 - 6 \times 7$

$(a+b)c = a\cdot c + b\cdot c$

$20 = 20$

1.1.6 Order (Inequalities) of Real Numbers

Properties	Examples
(i) Trichotomy Property $\forall\ a, b \in \mathbb{R}$ Exactly one of the following holds: • $a < b$ or $a > b$ or $a = b$	**Keep in mind!** $\forall$ stands for "for all" • $7 < 10$, but $7 \not> 10$ and $7 \neq 10$
(ii) Transitive Property $\forall\ a, b, c \in \mathbb{R}$ • If $a > b$ and $b > c$, then $a > c$ or • If $a < b$ and $b < c$, then $a < c$	• $8 > 5$ and $5 > 4 \Rightarrow 8 > 4$ or • $2 < 5$ and $5 < 6 \Rightarrow 2 < 6$
(iii) Addition Property $\forall\ a, b, c \in \mathbb{R}$ • If $a < b$, then $a + c < b + c$ or • If $a > b$, then $a + c > b + c$	• $5 < 7$ and $c = 2 \Rightarrow 5 + 2 < 7 + 2$ i.e., $7 < 9$ or • $10 > 8$ and $c = 2 \Rightarrow 10 + 2 > 8 + 2$ i.e., $12 > 10$
(iv) Subtraction Property $\forall\ a, b, c \in \mathbb{R}$ • If $a < b$, then $a - c < b - c$ or • If $a > b$, then $a - c > b - c$	• $10 < 11$ and $c = 2 \Rightarrow 10 - 2 < 11 - 2$ i.e., $8 < 9$ or • $13 > 9$ and $c = 2 \Rightarrow 13 - 2 > 9 - 2$ i.e., $11 > 7$
(v) Multiplication Property • If $a < b$ and $c > 0$, then $a\cdot c < b\cdot c$ or • If $a < b$ and $c < 0$, then $a\cdot c > b\cdot c$	• $5 < 8$ and $c = 2 \Rightarrow 5\cdot 2 < 8\cdot 2$ i.e., $10 < 16$ or • $5 < 8$ and $c = -2 \Rightarrow 5(-2) > 8(-2)$ i.e., $-10 > -16$

Try yourself!

Write the multiplicative property for $a > b$

Remember!

If we multiply or divide an inequality by a negative number, then the inequality sign is always changed.

Properties	Examples
(vi) Division Property	
If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$	$6 < 9$ and $c = 3 \Rightarrow \frac{6}{3} < \frac{9}{3}$ i.e., $2 < 3$
or	or
If $a < b$, and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$	$6 < 9$ and $c = -3 \Rightarrow \frac{6}{-3} > \frac{9}{-3}$ i.e., $-2 > -3$
Try yourself! Write the division property for a > b.	
(vii)	
If a and b both are positive (+) or both are negative (–), then $a < b \Leftrightarrow \frac{1}{a} > \frac{1}{b}$ or $a > b \Leftrightarrow \frac{1}{a} < \frac{1}{b}$	$2 < 4 \Rightarrow \frac{1}{2} > \frac{1}{4}$ or $-3 < -2 \Rightarrow -\frac{1}{3} > -\frac{1}{2}$